SPATIALLY COMPACT CLUTTER

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Clutter false alarms plague most active sonar systems. A common source of false alarms is examined in this paper—spatially compact clutter. Echoes from a shipwreck are analyzed and found to have clutter-producing heavy-tailed probability density functions and similar statistics at both 2-km and 10-km ranges, the latter in contrast to expectations based on the different propagation conditions. The clutter model of [1] is applied to represent spatially compact clutter, illustrating that clutter objects with minimal down-range extent appear to be minimally impacted by multipath, potentially explaining the similarity observed in the real data. The theory also indicates that when the clutter source is sufficiently small and strong relative to the surrounding background, the sonar receiver statistics are largely unaffected by variations in multipath, matched filtering, and array processing.

1 Introduction
Active sonar systems are typically plagued by a plethora of false alarms in shallow-water operational environments. The false alarms arise from a variety of both natural and man-made sources and can overload a sonar operator or down-stream automated signal and information processing routines. Physics-based statistical models of active sonar clutter exist [2, 3] and can account for the effects of propagation in a shallow-water environment and array processing [4, 5, 1], but assume that the clutter source is spatially distributed over an extent covering the multipath that contribute to the sonar response at a given delay time. However, an important class of clutter comprises spatially compact sources such as shipwrecks, small fish schools, or single rock escarpments or mud volcanoes.

In this paper a brief statistical analysis of echoes from a shipwreck is presented to motivate the ensuing theoretical analysis of the sonar receiver envelope statistics for spatially compact clutter. In particular, the effects of array processing and multipath propagation on clutter statistics are evaluated and discussed in comparison with the results of the shipwreck data analysis.

2 Statistics measured from a shipwreck
Echoes from a shipwreck at 36° 18.804′ north by 14° 41.136′ east (Malta Plateau region in the Mediterranean Sea) were measured during the NURC Clutter 07 experiment on May
The data were obtained during circular tracks about the shipwreck at 2- and 10-km ranges with a towed-array of triplet sensors. The shipwreck is of unknown origin, but noted to be approximately 65 m long [6]. After beamforming, matched filtering, and semi-automated echo extraction, a statistical analysis of the shipwreck echoes was performed for data in 100 Hz bands in two frequency regimes (800–1800 Hz and 2–3.5 kHz) arising from the simultaneous transmission of linear frequency modulated waveforms. The $K$-distribution [7] shape parameter ($\alpha$) was used to represent the severity of the envelope probability density function (PDF) tails with small values representing a high probability of false alarm ($P_{fa}$) and large values a Rayleigh-like PDF. Although not presented in this paper, the fit of the $K$ distribution to the envelope-PDF was examined and found to be adequate in the tails, so $\alpha$ is an acceptable proxy for false alarm performance.

Values of $\alpha$ estimated from the echoes are shown in Fig. 1 for the two frequency regimes and the 2- and 10-km ranges as a function of azimuth from the shipwreck, which can be related to the wreck’s aspect of ensonification. Nearly $360^\circ$ of azimuth were covered at a distance of 2 km and about $270^\circ$ at 10 km. The echoes exhibit highly non-Rayleigh character (i.e., small $\alpha$) for all azimuths as well as a strong similarity in $\alpha$ across both frequency regime and range for many azimuths, with the latter of primary interest in this paper. A scatter plot of the estimates is shown in Fig. 2 for the common azimuths of the two ranges where 60% of the 10-km-range data are seen to lie within a factor of two of the corresponding 2-km-range data (i.e., $\alpha_{10km}/2 \leq \alpha_{2km} \leq 2\alpha_{2km}$).

Based on the results of [4] and [5], the greater quantity of multipath (see Sect. 3.2 for the multipath structure of this environment) at the 10-km range should result in a larger $\alpha$ compared with that for the 2-km range. The theory leading to this expectation assumes that the clutter source extends over the region of the bottom spanned by the multipath contributing to clutter at a given delay time. Clearly the shipwreck does not satisfy this condition; however, as shown in the following section, the modeling results of [1] may be adapted to represent spatially compact clutter and perhaps explain the similarity of $\alpha_{10km}$ to $\alpha_{2km}$ observed in Figs. 1 and 2.

### 3 Spatially compact clutter modeling

Although the K distribution may not be sufficient to represent extremely heavy-tailed clutter (e.g., as might the log-normal or generalized gamma PDFs), it is adequate in the tails of the PDF for the shipwreck echoes presented in the previous section and may be acceptable for a broader class of spatially compact clutter. From [1], the $K$ distribution shape parameter of the clutter response at time $\tau$ after matched filtering and beamforming to bearing $\psi$ is

$$
\alpha(\tau, \psi) = \frac{\left[ \int_0^{2\pi} \int_0^\infty \beta_0(r, \phi) \lambda_0(r, \phi) M_\psi(r, \phi) r \, dr \, d\phi \right]^2}{2 \int_0^{2\pi} \int_0^\infty \beta_0(r, \phi) \lambda_0^2(r, \phi) M_\psi^2(r, \phi) r \, dr \, d\phi},
$$

where $\beta_0(r, \phi)$ and $\lambda_0(r, \phi)$ are the independent-scatterer spatial density and average power at horizontal range $r$ and bearing $\phi$, and

$$
M_\psi(r, \phi) = |\hat{c}_{\psi, \phi}(\tau \cos^2 \theta_g - \gamma r)|^2
$$
Figure 1. Shape parameter estimates at 2- and 10-km ranges and LF and MF bands displayed relative to shipwreck aspect.

Figure 2. Scatter plot of shape parameter estimates for the two ranges. Filled and un-filled symbols are for the MF and LF bands, respectively. Upright triangles (△) are for peak SNR > 30 dB at the 2-km range, inverted triangles (▽) for peak SNR < 25 dB, circles (◦) for the intervening regime.
where \( \theta_g \) is the grazing angle of the dominant path contributing to the clutter at range \( r \) and \( \gamma = 2 \cos \theta_g/c_w \) where \( c_w \) is the average speed of sound in water. The spatio-temporal template function \( \tilde{c}_{\psi,\phi}(t) \) (and \( M_{\psi}(r, \phi) \) through (2)) represents the effects of the transmit waveform, propagation, and array processing. Complex envelopes or basebanded spectra are denoted with a tilde (e.g., \( \tilde{c}_{\psi,\phi}(t) \)).

Spatially compact clutter may be characterized by appropriately defining \( \beta_0(r, \phi) \) and \( \lambda_0(r, \phi) \). For example, by setting \( \beta_0(r, \phi) \) small and \( \lambda_0(r, \phi) \) large over the region of the compact clutter source relative to that found in surrounding areas.

The spatio-temporal template function is the complex envelope of \( c_{\psi,\phi}(t) \), which, for long ranges relative to water depth, is defined in the frequency domain as 

\[
C_{\psi,\phi}(f) \approx P_s(f)P^1/2_C(f)B_{\psi}(\phi,f)H_E(f)
\]

where \( P_s(f) \) is the Fourier transform of the transmit waveform autocorrelation function (ACF), \( P_C(f) \) is the spectrum of the clutter source backscatter, \( B_{\psi}(\phi,f) \) is the unsquared array beampattern with main response axis \( \psi \) and a source arriving from azimuth \( \phi \), and

\[
H_E(f) = \sum_{i=1}^{n_p} a_i e^{-j2\pi f \Delta \tau_i}
\]

represents the effects of acoustic propagation with \( (a_i, \Delta \tau_i) \) the amplitude and delay offset for the \( i \)th of \( n_p \) multipath contributing to the received signal at time \( \tau \) as described in [5].

### 3.1 Effect of array processing

If a spatially compact clutter source is sufficiently more acoustically reflective than the surrounding region, the integrals in (1) over azimuth would approximately be limited to the region \( \psi \pm L_c/(2r_c) \), where \( \psi, r_c \) and \( L_c \) are, respectively, the bearing, range, and cross-range extent of the clutter source. If the range is large enough for the beampattern to be nearly constant over this interval, (1) simplifies to

\[
\alpha(\tau, \psi) \approx \frac{L_c}{2r_c} \left[ \int_0^{\infty} \beta_0(r, \psi)\lambda_0(r, \psi)M_{\psi}(r, \psi)r \, dr \right]^2
\]

where a distortionless response constraint in the beamforming (i.e., \( B_{\psi}(\psi, f) = 1 \)) implies that \( \alpha \) does not have any further dependence on the beampattern.

However, when the clutter echo is not significantly larger than the diffuse reverberation from the surrounding seabed and ambient noise, the noise gain of an array can play an indirect role. The beam-output power levels of the diffuse reverberation and ambient noise depend on the array processing and can potentially impact the statistics of the received signal if the clutter-to-background power ratio (CBR) becomes low enough. For example, when the background has a Rayleigh-distributed envelope, a CBR of 20 dB results in an 8% increase in \( \alpha \).

In the shipwreck data described in Sect. 2, the array beamwidth at the center of each frequency band is approximately 2.2°, which exceeds the maximum azimuthal spread of a 65-m long ship at 2-km range (1.9°) and at 10-km range (0.4°). Thus, coupled with the fact that echoes on most azimuths have CBR > 20 dB, it may be expected that the difference in the cross-range extent of the sonar resolution cell at the two ranges (induced
by beamforming) does not largely change the statistics of the clutter echoes between the two ranges.

3.2 Effect of multipath

In modeling reverberation or clutter for active sonar, multipath are defined as the paths connecting the sound transmitter, some part of a scattering object or boundary, and the sonar receiver. In evaluating the statistics of clutter, similar to reverberation power-level modeling, the paths of concern are strictly those that contribute at a fixed delay time $\tau$. This differs from the more common one-way propagation problem where all the paths connecting two fixed points are enumerated resulting in different delay times and what is often called the channel impulse response. Here the transmitter, receiver, and delay-time are fixed and the ‘connecting’ paths are enumerated resulting in different positions for the scattering patch. In (4), these locations are described as delay offsets $\Delta r_i = \gamma \Delta r_i$ (recall $\gamma = 2 \cos \theta_g / c_w$) for $i = 1, \ldots, n_p$. From (2)-(4), the spatio-temporal template will have the form

$$M_{\psi}(r,\phi) = \left| \sum_{i=1}^{n_p} a_i \tilde{g}_{\psi,\phi} \left( \tau \cos^2 \theta_g - \gamma (r - \Delta r_i) \right) \right|^2$$

where $\tilde{g}_{\psi,\phi}(t)$ is the inverse Fourier transform of $C_{\psi,\phi}(f)/H_E(f)$ after basebanding and represents the effects of the transmit waveform, array processing, and the spectrum of the clutter backscatter. Assuming a range large enough for the beamforming to not alter the clutter statistics (as described in the previous section) and assuming a narrow enough transmit band that the clutter-backscatter spectrum is flat, the template function becomes

$$M_{\psi}(r,\psi) = \left| \sum_{i=1}^{n_p} a_i \tilde{R}_s \left( \tau \cos^2 \theta_g - \gamma (r - \Delta r_i) \right) \right|^2$$

where $\tilde{R}_s(t)$ is the autocorrelation function of the basebanded transmit waveform.

In the development of [1], (1) was simplified by assuming that the clutter was locally isotropic (i.e., the clutter parameters $\beta_0(r, \phi)$ and $\lambda_0(r, \phi)$ are constant over the extent of $M_{\psi}(r, \phi)$) resulting in an integral over the full range of the multipath (i.e., each path contributed to the clutter response with proportional weight described by the path amplitudes $a_i$). However, if the clutter characterization varies within the extent of $M_{\psi}(r, \psi)$, then the paths will be weighted differently. For example, a very small but highly reflective clutter source might only excite a few of the paths contributing to the sonar response and dominate other paths having contributions from the weakly reflective surrounding area. Such a situation could cause the similarity in the shape parameters of the shipwreck echoes measured at different ranges as described in Sect. 2.

To examine this in more detail, one-way propagation data collected on a vertical array near the shipwreck were used to refine inputs to the CAPARAY model [9] which was then used to obtain the multipath impulse response for one-way propagation from the sonar transmitter to the shipwreck. Assuming a monostatic sonar configuration, two-way multipath are formed by convolving the one-way channel impulse response with itself, yielding the path amplitudes and delays $(a_i, \tau'_i)$ for $i = 1, \ldots, n_p$. The two-way multipath characterization is used to approximate the locations on the bottom $(r_i)$ contributing to clutter at
delay time $\tau'_1$ through the transformation

$$r_i \approx \frac{2\tau'_1 - \tau'_i}{\gamma}$$

yielding $\Delta \tau_i \approx \tau'_1 - \tau'_i$. This approximation requires significantly less computational effort than attempting to find the exact paths connecting the source, some part of the bottom, and the receiver with all paths having the same delay time and is adequate for the present analysis. The resulting multipath impulse responses for use in computing clutter statistics (i.e., $\sum_{i=1}^{n_p} a_i \delta (r - r_i)$) for the two ranges are shown in Fig. 3 where the 10-km range exhibits denser paths and greater spreading than the 2-km range (note that the multipath were computed for a 2.25-km range). These represent the weight applied to different parts of the bottom in forming the received signal at time $\tau'_1$. Note that in Fig. 3 the approximation $\gamma \approx 2/c_w$ was used for display purposes with $c_w$ taken as the depth-averaged sound speed; calculations in the remainder of the paper are based on the $\Delta \tau_i$.

Figure 3. Multipath impulse responses for clutter at delay times corresponding to bottom scattering at 2.25- and 10-km ranges.

Now suppose that a strong clutter source having independent scatterer spatial density $\beta_0$ and average power $\lambda_0$ exists over the range $L = (r_a, r_b)$ (i.e., it has down-range extent $L_d = r_b - r_a$). Let the background be similarly represented by independent scatterer density $\beta_1$ and average power $\lambda_1$. The two-component sea floor, when inserted into (5), results in the shape parameter

$$\alpha(\tau, \psi) \approx \frac{L_c \beta_0}{2r_c} \left[ \int_L M_\psi(r, \psi) r \, dr + \frac{1}{8\pi} \int_{L_+} M_\psi(r, \psi) r \, dr \right]^2$$

$$+ \frac{\beta_1}{\beta_0 s^2} \int_{L_+} M_\psi^2(r, \psi) r \, dr.$$

(9)
SPATIALLY COMPACT CLUTTER

where \( L_\perp = \{(0, r_a) \cup (r_b, \infty)\} \) and \( s_c = \beta_0 \lambda_0 / (\beta_1 \lambda_1) \) is the independent-scatterer-level CBR given equal area. The background will typically have a lower power and lighter envelope-PDF tails than the clutter, implying \( \alpha_1 > \alpha_0 \) and \( s_c \gg 1 \). For very strong clutter objects, this simplifies to

\[
\alpha(\tau, \psi) \approx \frac{L_c \beta_0}{2r_c} \left[ \int_{r_a}^{r_b} M_\psi(r, \psi) r \, dr \right]^2
\]

and when the down-range extent of the clutter object is less than the transmit waveform resolution, \( L_d = r_b - r_a < c_w/(2W) \), the shape parameter only depends on the size of the clutter object and its independent scatterer density,

\[
\alpha(\tau, \psi) \approx \frac{\beta_0}{2} L_c L_d.
\]

This result indicates that when the clutter object is small enough in both range and cross-range, the statistics of the sonar response are unaffected by variations in multipath, matched filtering, and array processing.

The effect of the multipath shown in Fig. 3 on the shape parameter of the clutter received by the sonar for a 100-Hz bandwidth LFM transmit waveform is shown in Fig. 4 using (7) in (10) with normalization by \( L_c \beta_0 / 2 \). The ordinate in the figure can be interpreted as the effective down-range contribution length of the clutter source to \( \alpha \) as a function of its actual down-range extent taking into account the effects of propagation. The results illustrate that short clutter objects (relative to the waveform resolution and multipath spreading) contribute to \( \alpha \) proportionally with their down-range extent with little impact from varying propagation conditions. However, when the clutter source extends beyond the multipath spreading, its contribution to \( \alpha \) is capped with the limiting value dependent on the multipath. Comparing the two different ranges, the increased number of paths contributing at 10 km results in a larger overall shape parameter for large clutter sources as predicted in [5], but exhibits minimal difference for shorter clutter objects, potentially explaining the similarity in \( \alpha \) observed in the shipwreck data shown in Sect. 2. As seen in Fig. 5, similar results are obtained when (9) is implemented with a 25 dB CBR at 2 km and an 18 dB CBR at 10 km. For very short clutter objects, the background is seen to dominate and cause an increase in the ordinate over direct proportionality with \( L_d \) indicating an increase in \( \alpha \). For this example, the clutter and background were assumed to have the same independent scatterer density (i.e., \( \beta_0 = \beta_1 \)); however, as seen in (9), \( \alpha \) is only very weakly dependent on the ratio \( \beta_0 / \beta_1 \) when \( s_c \) is reasonably large.

4 Conclusions

A preliminary statistical analysis of echoes from a shipwreck indicated that they are heavy-tailed (i.e., false alarm inducing) with \( K \)-distribution shape parameters (\( \alpha \)) that were small (mostly within \( 0.5 - 3 \)) and varying with the aspect of ensonification. A strong similarity in \( \alpha \) was observed over two frequency bands and at both 2- and 10-km ranges for a given azimuth.

The clutter modeling of [1] was applied to spatially compact clutter to explain the results of the shipwreck analysis. As might have been expected, when the clutter object is
wholly within the mainlobe of the array beampattern, the array processing had no significant impact on the clutter statistics. Potentially explaining the similarity in $\alpha$ observed at multiple ranges in the shipwreck data, theoretical predictions for the clutter statistics were observed to be insensitive to multipath when the clutter object had minimal down-range extent. When the clutter source was sufficiently small, the echo statistics were predicted to be unaffected by variations in multipath, matched filtering, and array processing.

These results may play an important role in the inversion of sonar data for clutter source parameters, clutter simulation, and the development of classification algorithms attempting to separate targets from clutter.

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References

Figure 4. Effect of multipath on clutter statistics as a function of clutter source down-range extent ($L_d$) at 2- and 10-km ranges for an infinite CBR.

Figure 5. Effect of multipath on clutter statistics as a function of clutter source down-range extent ($L_d$) at 2- and 10-km ranges with CBR of 25 dB at 2 km and 18 dB at 10 km.